Math 7760 − Homework 4 − Due: October 3, 2022

Practice Problems:

Problem 1. Let r, n be nonnegative integers with $r \leq n$. Let E be an n-element set and define $\mathcal{B}$ to be the set of all r-element subsets of E.

    (1) Convince yourself that $\mathcal{B}$ is the set of bases of a matroid $U_{r,n}$. Matroids of this form are called *uniform matroids.*

    (2) Determine what the independent sets, circuits, rank function, closure operator, and spanning sets of this matroid are.

Problem 2. Oxley, section 1.1 problems 1 and 4.

Problems to write up:

Problem 3. Prove that $U_{2,n}$ is representable over a field with q elements if and only if $q \geq n-1$. Does there exist a graphic matroid that is not representable over $\mathbb{F}_2$? Prove your answer.

Problem 4. Let M be a binary matroid, i.e. a matroid representable over the field with two elements. Prove that given any distinct circuits C_1, C_2, their symmetric difference $C_1 \Delta C_2$ contains a circuit.